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AUTHOR Rosthal, Richard A.
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ABSTRACT

Four measures of disparity were used to compute changes in the distribution of education dollars for each of the 50 states between 1970 and 1975. Three of the measures have been used by analysts and have known statistical properties, and the fourth is a less formal measure that has been embodied in a federal regulation. These measures include 1) the coefficient of deviation, 2) the coefficient of variation, 3) the Gini index, and 4) the 95:5 ratio. To test the expectation that the results obtained from the four measures would differ, all four measures were calculated for identical data bases--the distribution of state and local shares of current operating expenditures for 1970 and 1975, using data produced by the National Center for Education Statistics. The degree of similarity can be assessed by calculating the correlation coefficients between each pair of measures. The three statistical measures are virtually identical in result, with correlations ranging from .94 to .98. Even the 95:5 ratio has a high correlation with statistical measures. (Author/JG)

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Measures of Disparity

A Note

Richard A. Rosthal

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903 South Highland Street
Arlington, Virginia 22204
(703) 892-4500

KILLALEA ASSOCIATES, Incorporated

Introduction

A number of measures of disparity are available for use in studies of within-state distribution of education resources, and it is necessary to select among them. Of course, if only one measure has properties that make it peculiarly well suited to the purposes of an analysis -- that is, the kind of disparity that is of most interest is much better captured by one measure than by others -- then this purpose will dictate the choice. Usually, this is not the case, and several measures seem equally applicable to the problem to be analyzed. In such situations, it is important to know how their results differ. To the extent that they differ, two courses are open in presenting and interpreting results. One is to employ one measure, but then the interpretation of results must be accompanied by provisos about their uncertainty. The other is to present the results for all reasonably relevant measures; but, unavoidably, the presentation and interpretation are much more complicated. Needless to say, it is distinctly preferable if the results for all relevant measures turn out to be very similar.

This problem arose in a study, recently completed, of changes in the distribution of education dollars between 1970 and 1975.* This note is being circulated because of the interest other analysts may have in the way the problem was resolved.

Comparison of Measures

Four measures were computed for each of the two years, 1970 and 1975. Three have been in use by analysts and have known statistical properties. The fourth is a less formal measure that has been embodied in a federal regulation.

The coefficient of deviation is the average deviation from the mean divided by the mean. It weights each student's spending in proportion to its distance from the mean. Thus, a student receiving $40 more

*The Effects of School Finance Reform: Changes from 1970 to 1975, J. N. Killalea and R. A. Rosthal, Killalea Associates, Incorporated, December 1977.

than the mean contributes as much to the coefficient of deviation as two students receiving $20 more than the mean.

The coefficient of variation is the standard deviation of a distribution divided by its mean. In applying it to a distribution, each student's spending level is weighted as the square of its distance from the mean. In contrast to the coefficient of variation, a student receiving $40 more than the mean contributes as much to the coefficient of variation as four students receiving $20 more than the mean.

The Gini index is most easily understood by considering the Lorenz curve, which graphs the percentage of total funds received by the lowest x percent of the students, versus x. For a perfectly even distribution, the curve is a straight line. For other distributions, it is a curve underneath the straight line. The Gini index is the fraction of the total area under the straight line that lies between the line and the curve. It ranges between zero and one, being zero in the case of perfect equality and approaching one as all funds are concentrated on fewer students. It is not possible to state how the Gini index weights different parts of a distribution since it effectively compares all parts of the distribution to all other parts. It is possible to say, however, that for a wide range of distributions it is approximately half the value of the coefficient of variation; but if only a small number of students receive either more or less than the rest, the Gini index can be much smaller than the coefficient of variation.

The 95:5 ratio is the ratio of spending at the 95th percentile of students to spending at the 5th percentile. It compares pupils at only these two points in the distribution and so is not sensitive to variations in a large part of the distribution. It is included here because it is simple to understand and because it is used in a federal regulation.*

The coefficient of deviation and coefficient of variation are structurally related, but the Gini index is constructed quite differently. And, of course, the 95:5 ratio is different from all three

*45 CFR 115.63.

statistical measures. It therefore might be expected that the results obtained from the four measures would also differ. To test the expectation, all four measures were calculated for identical data bases -- the distribution of state and local share of current operating expenditures for 1970 and 1975, using the Census/ELSEGIS data bases produced by the National Center for Education Statistics. The results are shown in Tables I and II, for the two years, respectively.

A quick review of the tables indicates that the measures rank the states in ways that are at least somewhat similar. But it is difficult to see how similar they are, since their results are expressed in different absolute numbers. The degree of similarity can be assessed by calculating the correlation coefficients between each pair of measures. The correlations are shown in the tabulations below (Hawaii is omitted because it is essentially a single school district and therefore was treated in the referenced study as having no disparity).

1970

	95:5	Coefficient of Deviation	Coefficient of Variation	Gini
95:5	1	.84	.89	.89
Coefficient of Deviation		1	.94	.98
Coefficient of Variation			1	.96
Gini				1

1975

	95:5	Coefficient of Deviation	Coefficient of Variation	Gini
95:5	1	.83	.85	.89
Coefficient of Deviation		1	.95	.98
Coefficient of Variation			1	.97
Gini				1

The three statistical measures are virtually identical in result, with correlations ranging from .94 to .98. Even the 95:5 ratio has a high correlation with the statistical measures. We conclude that, for the purposes of the study (and for many others as well) any one of the

statistical measures is acceptable, and moreover that even the 95:5 ratio is reasonable in case one wished to use it because of its simplicity.

The four measures described above are symmetric in the sense that each treats in the same way the upper and lower parts of the distribution. For some purposes one might wish instead to consider a measure that weights heavily the lower spending end of a distribution and weights more lightly the higher spending end. In the course of estimating the costs of equalization in the referenced study, one simple measure of this sort was developed. Equalization costs were calculated for each state to bring all low-spending districts up to some given level. For instance, What does it cost a state to achieve a 95:5 ratio of 1.40 by increasing spending in the lower spending districts (while leaving higher spending districts at their current levels)? If this cost is expressed as a percentage of total spending in a state, one has a simple disparity measure that weights students below the stated level in proportion to their distance from the stipulated level and ignores students above that level. Results of applying that measure are presented in the referenced report. Because the purpose of this measure is different from that of the four measures already reviewed, it is to be expected that it would rank states differently. Indeed, this does occur, but not as much as might have been thought. The correlation of this measure with the three statistical measures varies from .73 to .75, and its correlation with the 95:5 ratio is .84.

Table I. Within-State Disparities for Four Measures, 1970

	95:5 Percentile	Coefficient of Deviation	Coefficient of Variation	Gini Index
Alabama	1.43	0.09	0.12	0.07
Alaska	1.30	0.05	0.08	0.04
Arizona	1.88	0.17	0.21	0.12
Arkansas	1.96	0.14	0.18	0.10
California	1.86	0.15	0.21	0.11
Colorado	1.76	0.13	0.16	0.09
Connecticut	2.22	0.18	0.23	0.12
Delaware	2.06	0.12	0.16	0.09
Florida	1.53	0.11	0.13	0.07
Georgia	1.86	0.15	0.20	0.10
Hawaii	1.00	0.00	0.00	0.00
Idaho	1.49	0.09	0.12	0.07
Illinois	2.05	0.18	0.23	0.12
Indiana	1.59	0.11	0.14	0.08
Iowa	1.82	0.13	0.18	0.09
Kansas	1.86	0.13	0.22	0.10
Kentucky	1.72	0.15	0.17	0.10
Louisiana	1.33	0.08	0.09	0.05
Maine	1.57	0.12	0.16	0.09
Maryland	1.60	0.10	0.14	0.07
Massachusetts	1.94	0.15	0.19	0.10
Michigan	1.81	0.14	0.19	0.10
Minnesota	1.46	0.10	0.13	0.07
Mississippi	1.71	0.14	0.17	0.09
Missouri	1.84	0.16	0.25	0.11
Montana	1.91	0.16	0.19	0.11
Nebraska	1.63	0.09	0.15	0.07
Nevada	1.19	0.06	0.08	0.03
New Hampshire	1.88	0.12	0.17	0.09
New Jersey	1.93	0.15	0.20	0.11
New Mexico	1.51	0.09	0.14	0.07
New York	1.64	0.13	0.16	0.09
North Carolina	1.49	0.09	0.12	0.07
North Dakota	1.95	0.13	0.17	0.09
Ohio	1.78	0.16	0.19	0.11
Oklahoma	1.67	0.15	0.20	0.10
Oregon	1.50	0.09	0.12	0.06
Pennsylvania	1.57	0.13	0.16	0.09
Rhode Island	1.75	0.15	0.19	0.10
South Carolina	1.66	0.11	0.13	0.08
South Dakota	1.71	0.11	0.18	0.08
Tennessee	1.91	0.21	0.23	0.13
Texas	1.91	0.15	0.23	0.11
Utah	1.27	0.06	0.09	0.04
Vermont	3.36	0.26	0.38	0.19
Virginia	1.66	0.17	0.22	0.11
Washington	1.66	0.14	0.17	0.10
West Virginia	1.49	0.11	0.14	0.08
Wisconsin	1.60	0.11	0.16	0.08
Wyoming	1.57	0.13	0.17	0.09

Table II. Within-State Disparities for Four Measures, 1975

	95:5 Percentile	Coefficient of Deviation	Coefficient of Variation	Gini Index
Alabama	1.43	0.10	0.12	0.07
Alaska	1.29	0.11	0.16	0.07
Arizona	1.71	0.13	0.17	0.09
Arizona	1.78	0.14	0.18	0.10
California	2.02	0.15	0.21	0.11
Colorado	1.77	0.13	0.18	0.10
Connecticut	2.29	0.16	0.21	0.12
Delaware	1.70	0.14	0.18	0.09
Florida	1.30	0.07	0.09	0.05
Georgia	2.41	0.21	0.28	0.15
Hawaii	1.00	0.00	0.00	0.00
Idaho	1.51	0.13	0.16	0.09
Illinois	1.90	0.18	0.22	0.12
Indiana	1.50	0.11	0.13	0.07
Iowa	1.34	0.07	0.09	0.05
Kansas	1.65	0.09	0.14	0.07
Kentucky	1.86	0.17	0.20	0.11
Louisiana	1.32	0.08	0.10	0.05
Maine	1.67	0.13	0.16	0.09
Maryland	1.77	0.15	0.20	0.10
Massachusetts	2.17	0.17	0.23	0.12
Michigan	1.71	0.12	0.17	0.09
Minnesota	1.62	0.11	0.15	0.08
Mississippi	1.80	0.14	0.17	0.09
Missouri	1.73	0.16	0.24	0.11
Montana	1.97	0.15	0.21	0.11
Nebraska	1.73	0.12	0.19	0.09
Nevada	1.18	0.04	0.07	0.03
New Hampshire	1.78	0.12	0.16	0.09
New Jersey	1.95	0.16	0.20	0.11
New Mexico	1.41	0.08	0.13	0.06
New York	1.85	0.20	0.23	0.13
North Carolina	1.51	0.09	0.12	0.07
North Dakota	1.53	0.11	0.14	0.08
Ohio	1.78	0.16	0.20	0.11
Oklahoma	1.51	0.13	0.20	0.09
Oregon	1.50	0.11	0.14	0.08
Pennsylvania	1.57	0.14	0.17	0.09
Rhode Island	1.58	0.11	0.13	0.08
South Carolina	1.65	0.10	0.14	0.08
South Dakota	1.50	0.09	0.13	0.07
Tennessee	1.90	0.18	0.21	0.12
Texas	1.79	0.14	0.20	0.10
Utah	1.27	0.07	0.09	0.04
Vermont	1.99	0.18	0.21	0.12
Virginia	1.78	0.21	0.27	0.14
Washington	1.83	0.14	0.18	0.10
West Virginia	1.49	0.10	0.13	0.07
Wisconsin	1.59	0.12	0.16	0.09
Wyoming	1.82	0.15	0.21	0.10